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ABSTRACT

Harter, Richard M., M.S., Purdue University, August, 1989. An Estimation of Rainfall Amounts Using Radar-Derived Z-R Relationships. Major Professor: John T. Snow.

Directly-measured reflectivity values (Z) by radar and rainfall rate values (R) by rain gages are correlated over five-minute intervals to a maximum of one hour for 260 rain gage points. A non-linear regression of the expression $R = aZ^b$ is performed, with Z the independent variable. This expression is solved for Z to obtain Z-R relationships of the form $Z = AR^B$ for each five minute period. A median value of the coefficient A and the exponent B is computed to represent the complete hour. Z-R relationships are then used to calculate estimates of average rainfall amount over a rain gage area. These radar-estimated values of average rainfall amount are then compared to the average rainfall amounts measured by the rain gages for dependent as well as independent data sets. The significant effect of hail is also examined. Results indicate that directly-measured Z-R relationships derived from data containing shallow precipitation gradients were the most accurate in estimating the average water amount for the rain gage network area. In contrast, Z-R relationships derived from data containing steeper gradients of precipitation were not as accurate in estimating the average rainfall amount. The Z-R relationships of Marshall-Palmer, Jones, and the NEXRAD relationship underestimated the average rainfall amount by almost 30% for the data set containing shallow gradients of preicpitation. All Z-R relationships overestimated the amount of rainfall in the network for data sets

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AN ESTIMATION OF RAINFALL AMOUNTS USING RADAR-DERIVED Z-R RELATIONSHIPS

A Thesis

Submitted to the Faculty

of

Purdue University

by

Richard M. Harter

In Partial Fulfillment of the

Requirements for the Degree

of

Master of Science

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containing hail. Once the hail was "removed", Z-R rainfall estimates improved significantly.

1. INTRODUCTION

Weather radar has been used experimentally to measure rainfall for nearly 40 years. There has been considerable operational interest in this technology, since it provides spatially and temporally continuous rainfall measurements that are immediately available at one location. This is desirable since long-term maintenance of dense rain gage networks is not economically feasible. Therefore, radar has potential to improve operational forecasting of river flow and flash floods, thereby saving lives and property.

Fundamental to any measurement of precipitation by means of radar is a relationship between radar reflectivity and rainfall rate. Both parameters are a function of the raindrop size distribution, N(D), which is the number of drops per unit volume.

The drop size distribution is described in terms of the distribution function:

$$N(D) = \int_0^\infty n(D)dD, \quad (1)$$

where D is the drop diameter, and

n(D) is the number of drops per unit volume with diameters between D and D+dD.

It is the frequency distribution of drop sizes that is characteristic of a given cloud or a given rainfall. In convective-type precipitation, the drop-size distribution is found to vary with height and to change with time.

Most work in estimating rainfall from radar measurements has been directed toward establishing a relationship between reflectivity Z and rainfall rate R, where

$$Z = \int_0^\infty n(D) D^6 dD, \tag{2}$$

$$R = \int_0^\infty \pi/6 \; n(D) D^3 V_t(D) dD, \text{ and} \tag{3}$$

$V_t(D)$ is the terminal velocity of a drop with diameter D.

This relationship is generally reported in the form

$$Z = AR^B, \tag{4}$$

where A and B are parameters to be determined.

Equations 3 and 4 show that both reflectivity and the rainfall rate are directly proportional to the drop-size distribution. This results in Z being proportional to the sixth power of the drop diameter. Equation 4 suggests that the rainfall rate is proportional to the third power of the drop diameter; however, rainfall rate also depends on the drop terminal velocity, which is as yet an unspecified function of D. Table 1 (Mueller et al., 1967) shows mean rainfall rates (in units of mm/hr) as a function of radar reflectivity (in units of mm^6/m^3) for different geographical locations. In general, as radar reflectivity increases, so does the rainfall rate. Therefore, this data, along with equations 3 and 4, show a strong dependency of both reflectivity and rainfall rate on drop size. Since the operational meteorologist wishes to predict a rainfall rate from a measurement of radar reflectivity, reflectivity should be treated as the independent variable. Therefore, in this study, Z will be the independent variable.

1.1 Indirect Method of Measuring Z and R

The indirect method of determining a relationship between radar reflectivity and rainfall rate is through measurement of the drop-size distribution. The most serious difficulty with this type of measurement is that the volume in space

Table 1

Mean Rainfall Rates as a Function of
Reflectivity for Different Geographical Locations

Radar Reflectivity mm^6/m^3	Rainfall Rate (mm/hr) Florida	Marshall Islands	Oregon	Indonesia	Alaska	North Carolina
1.1×10^2	1.0	1.0	0.6	0.5	0.6	0.6
3.5×10^2	1.0	1.6	1.1	1.1	1.2	1.6
1.1×10^3	2.5	3.7	2.3	2.4	2.8	3.5
3.5×10^3	6.3	8..7	5.4	6.0	5.2	7.8
1.1×10^4	14.5	21.6	9.5	14.4	8.8	17.7
3.5×10^4	34.8	48.4	18.7	29.5	9.0	38.7
1.1×10^5	68.5	90.5		65.7	9.2	87.1
3.5×10^5	167.1			70.0		
1.1×10^6	247.7			123.8		

in which drops can be sampled is limited to at most a few cubic meters. The assumption must then be made that these few cubic meters are representative of the 10^5 or 10^6 cubic meters sampled by a radar. Mueller and Sims (1966) showed that for a sample at 5 ft. above ground, a drop-size distribution measured over 44 m^3 is required to estimate the rainfall rate to within 10 percent with 95 percent confidence.

To determine the rainfall rate from drop-size distributions requires knowledge of the velocity of the individual raindrops. The raindrop terminal velocity reported by Gunn and Kinzer (1949) is widely accepted. This relationship is reasonable near the ground since there is generally good agreement between the average rainfall rates computed from drop-size distributions and rates measured by rain gages. However, further aloft where the radar is measuring, raindrop terminal velocities are affected by updrafts and downdrafts and the Gunn-Kinzer relationship may no longer hold.

Therefore, using this indirect method, if one knows the drop-size distribution and the raindrop terminal velocities, reflectivity and rainfall rate can be determined. Then, by equating Z and R, a Z-R relationship can be determined. Marshall and Palmer (1948) were the first of several investigators that used drop-size spectra and drop terminal velocity to obtain a Z-R relationship. (Appendix A shows a mathematical determination of the Z-R relationship for a Marshall-Palmer (exponential) drop-size distribution and an assumed raindrop terminal velocity.) The resulting relationship, $Z=200\ R^{1.6}$, has become the standard for stratiform precipitation.

In Hawaii, Blanchard (1953) measured drop-size distributions for non-orographic rain, orographic rain at cloud base, and orographic rain within a cloud. Three different Z-R relationships were derived. This work was important because it showed the effect of the drop-size distribution on the reflectivity factor, Z. Blanchard found lower coefficients in the Z-R relationship for a drop distribution that was skewed toward the smaller drop sizes.

The importance of the drop-size distribution continued to be emphasized in the 1960's. In order to improve the accuracy of rainfall measurements by radar, a large variety of drop-size distributions among different storms and within an individual storm were investigated by Fujiwara (1965). This study provided a more comprehensive and continuously varying relationship between radar reflectivity and rainfall rate, dependent on the type of storm.

Zoote (1966), like Blanchard, took observations of rain drop spectra close to the bases of showers and thunderstorms. However, the observing site in Tuscon was at an elevation of 8200 ft. while cloud bases were between 8000 to 10,000 ft. Observations close to the cloud base (or in some cases, within the base of the clouds) helped minimize the change in the drop-size distribution observed at cloud base due to drop collision, evaporation, gravity separation and wind shear. The largest drops measured by Blanchard seldom exceeded 2 mm in diameter, while some Tuscon samples had drops as large as 5 to 6 mm. Most drops, however, averaged 3 to 4 mm. Zoote's drop-size distribution was, therefore, much broader than Blanchard's and yielded a higher coefficient and exponent in the Z-R relationship.

The relationships determined by Mueller were deduced from drop-size distributions obtained from a raindrop camera. The camera photographed raindrops in a 1 m^3 volume for 10 seconds. Samples were taken once per minute. The drop-size distributions were obtained, and then the Z-R relationships calculated using a logarithmic least-squares fitting technique. A $Z=372\ R^{1.47}$ relationship for Illinois was among those found.

Table 2 (Mueller, et al., 1967) lists Z-R relationships as determined from drop-size spectra from a number of different investigators. Examination of Table 2 shows a wide variation in both the coefficient A and the exponent B in the Z-R relationships. Some of the variation results from the different techniques used to measure the drop-size distributions. But wherever the same technique has been used, considerable differences exist due to topography, geographical variation, rain type, synoptic type, and the thermodynamic structure of the atmosphere.

The increase of the exponent B from continuous rain to thunderstorms indicates that the more showery a rain becomes, the higher the reflectivity for medium to high rain rates (implying the presence of larger raindrops).

As can be seen, the Z-R relationship is not unique. Physical mechanisms that may alter the drop-size distribution, such as evaporation and coalescence, are listed in Table 3 (Wilson and Brandes, 1979) with an indication of their probable influence on the Z-R relationship and the storm region where the effect is probably at a maximum. Perhaps a more efficient method to correlate the reflectivity, Z, with the rain rate, R, is to measure both quantities directly.

1.2 Direct Method of Measuring Z and R

A more direct method of relating Z and R is by simultaneous measurement of both quantities. Rain gages directly measure rainfall rate while a radar directly measures reflected power. The equation that describes this latter measurement is the "radar equation":

$$P_r = c|K|^2 Z/r^2, \tag{5}$$

where

Table 2

Radar Reflectivity Rainfall Rate Relationships
from Drop Size Spectra

Investigator	$Z = AR^B$ A	B	Standard Error of Estimate of log R	Comments
Marshall, J.S. (1947)	220	1.6		Widely accepted and used
Blanchard, D.C. (1953)	31	1.71		Orographic Hawaiian rain at cloud base
	16.6	1.55		Orographic Hawaiian rain within the cloud
Fujiwara, M. (1967)	80	1.38		Orographic Hawaiian rain
Hardy, K.R. (1962)	312	1.36		Arizona and Michigan rain with rates greater than 5 mm/hr
Imai (in Japan) (1960)	700	1.6		One day of probably warm rain
	300	1.6		One day of probably warm rain
	300	1.6		One day continuous rain
	200	1.5		Air Mass showers
	80	1.5		Pre-warm front rain
Diem, M. (1966)	184	1.28		Overall average of different locations
	278	1.30		Entebbe Uganda (tropical)
	240	1.30		Lwin Congo (tropical)
	176	1.18		Palma
	151	1.36		Barza, Italy
	179	1.25		Karlsruhe, Germany - spring
	227	1.31		Karlsruhe, Germany - summer
	178	1.25		Karlsruhe, Germany - fall
	150	1.23		Karlsruhe, Germany - winter
	137	1.36		Axel Heiberg land
Foote, G.B. (1966)	520	1.81		Tucson, Arizona

Table 2, continued

Investigator	$Z = AR^B$ A	B	Standard Error of Estimate of log R	Comments
Dumoulin, G., & Gogolombles, A. (1966)	730	1.55		France, Average of all observations, 0.95 correlation coefficient
	255	1.45		
	426	1.5		
Mueller, E.A. (1967)	286	1.43	0.198	Florida
	221	1.32	0.170	Marshall Islands
	301	1.64	0.136	Oregon
	311	1.44	0.147	Indonesia
	267	1.54	0.142	Alaska
	230	1.40	0.171	North Carolina
	372	1.47	0.153	Illinois
	593	1.61	0.175	Arizona
	256	1.41	0.163	New Jersey

Table 3

Microphysical and Kinematic Influences on Z-R Relationships and the Effect on Radar Rainfall Estimates when no Adjustment is Applied.

Process	Change in $Z = AR^B$: A	Change in $Z = AR^B$: B	Probable effect on radar rainfall if Z-R is not adjusted	Possible region of maximum influence
Microphysical				
Evaporation (Atlas and Chmela, 1957)	Increase	Decrease	Overestimate	Inflow regions, fringe areas
Accretion of cloud particles (Atlas and Chmela, 1957; Rigby et al., 1954)	Decrease	Increase	Underestimate	Downdraft
Collision, coalescence (Srivastava, 1971)	Increase	Decrease	Overestimate	Reflectivity core
Breakup (Srivastava, 1971)	Decrease	Decrease	Underestimate	Reflectivity core
Kinematic				
Size sorting (Gunn and Marshall, 1955; Atlas and Chmela, 1957)	Increase	Decrease	Tendency to overestimate	Regions of strong inflow and outflow
Vertical Motion				
Updraft	Increase	Decrease	Overestimate	
Downdraft	Decrease	Increase	Underestimate	

P_r = average power returned to the radar receiver by the scatterers in a volume defined by the horizontal and vertical dimensions of the radar beam and the width of the radar pulse,

c = constant depending on the characteristics of the radar,

K = refractive index parameter, and

r = range of the target.

See Appendix B for development of the radar equation.

There are a number of disadvantages in the direct measurement technique. The primary one is that the radar samples rain aloft while the rain gage samples rain at surface. Austin and Williams (1951) attempted to minimize this problem by directing the radar beam directly over a rain gage located on a high point of ground. The radar beam was directed as close above the rain gage as possible without introducing ground return at the range of the gage. Since this ideal scanning method is rarely possible, investigators have attempted to time lag the radar observations to compensate for the time of fall of the raindrops. For example, if Z is measured during a certain five-minute period by radar, these values will be correlated with R values registered by rain gages for the next five-minute period.

Another disadvantage is the large difference in the volumes sampled by a radar and a rain gage. Assuming typical radar parameters of 1° horizontal beam width and 1 microsecond pulse width, the area over which the radar samples at a range of 10 km is about $2.6 \times 10\ m^2$. However, the rain gage samples an area on the order of $7 \times 10^{-2}\ m^2$. With increasing range, the radar sampling area becomes much larger. Dimaksyan, Zotimov and Zykov (1962) tried to reduce this area-sampling discrepancy by using more than one rain gage under a radar volume. They used three rain gage networks at ranges of 12, 22, and 32 km with 5, 9, and

12 gages all located in their respective radar areas. This configuration of gages yielded an extremely high gage density of one gage per 0.04, 0.045 and 0.05 km^2, respectively. Therefore, the radar measurement of Z could be correlated to the average rainfall rate of all gages in a particular area. Dimaksyan and Zotimov (1965) claimed that radar measurement of precipitation in an area would be more accurate than that obtained by a rain gage network of any density.

A third problem associated with an elevated radar sample is the horizontal drift of the raindrops during their fall from radar scanning location to the ground. This can be especially true in strong low-level wind conditions. In order to reduce this problem rain gage networks have been used by many investigators.

Despite the problems involved in direct measurement of reflectivity by radar, investigators have used this technique to estimate rainfall. Table 4 (Mueller, et. al., 1967) shows some Z-R relationships derived from direct measurement. Generally, the exponent B decreases as the coefficient A increases. As early as 1954, Hitschfield and Borden (1954) suggested that radar precipitation estimates should be calibrated against rain gages. In fact, the most successful technique for improving radar rainfall estimates has been to calibrate the radar with rain gages. Simple techniques that combine sparse gage reports (one gage per 1000-2000 km^2) with radar produce smaller measurement errors (10-30%) than with either gages or radar alone. However, the present density of rain gages across the U.S. of roughly 1 gage per 1000 square miles is not sufficient to obtain the accuracy in rainfall that a radar can provide.

Attempts to improve radar rainfall estimates by using rain gage "ground truth" values increased in the 1960's and 1970's. Jones (1964) used a network of 49 recording rain gages in a 400-square-mile area in East Central Illinois to study

Table 4

Radar Rainfall Relationships from Direct Measurement

Investigator	Geographical Location	Range of Applicability	$Z = AR^B$ A	B	Accurary Estimate (Standard Deviation)	Comments
Doherty, L.H. (1963)	Ottawa Canada	TRW	70	1.42	2.5 db	
		not TRW	38.4	1.63	1.7 db	
		R<10 mm/hr	18.6	2.37	1.6 db	
		R<20	25.9	2.02	1.7 db	
		R<40	33.9	1.79	1.9 db	
		R<60	38.2	1.69	1.9 db	
Berjuljew, G.P., Beznis A.M., et.al. (1963)	Valday USSR		340	1.5		The exponent is assumed equal to 1.5 and the coefficient determined from 2 years of rainfall.
Wilson, J.W. (1963)	Norman Okla.	TRW	45	1.43		Extreme low coefficient
		TRW	241	1.45		Extreme large coefficient
		TRW	183	1.18		Extreme low exponent
		TRW	141	1.72		Extreme high exponent
Aoyagi, J. (1964)	Tokyo		1000	1.4		For diffuse radar echoes

the accuracy of radar measurements of rainfall in 13 storms. He found that the radar overestimated the rainfall over the area in all but two storms. Jones used three Z-R relationships on the 1964 storms in East Central Illinois. For thunderstorms he used $Z = 435\ R^{1.48}$, for rainshowers, $Z = 370\ R^{1.31}$, and for stratiform rain, $Z = 311\ R^{1.43}$. By measuring the reflectivity factor Z with the radar, he calculated the rainfall rate R from one of the above Z-R relationships and then checked the calculated values of R with the rain gage value of R. Any differences between the two could, in theory, be rectified by adjusting the radar parameters in the radar equation to obtain a more accurate Z value and a better estimation of rainfall rate over the area by the radar. Jones concluded that it is not possible to know the exact Z-R relationship for each storm and within the same storm because of the physical processes such as evaporation and coalescence effecting the drop-size distribution.

Wilson (1970) used data from a 1100-square-mile rain gage network to obtain Z-R relationships for a number of thunderstorms in Oklahoma. He obtained the best Z-R relationship between the network average amounts from the radar and the network average amounts from the rain gages. In 4 of the 6 storms analyzed, his Z-R relationships did not depart significantly in terms of his measurement error from the Marshall-Palmer relationship of $Z = 200R^{1.6}$.

One might expect the need for more gages in highly-variable, convective-type rainfall. Huff (1970) used data from two dense rain gage networks in Illinois to investigate sampling errors in the measurement of areal rainfall for areas of 50 to 550 mi^2. The density of the rain gage network ranged from one per 25 mi^2 to one per 400 mi^2. He found that in the warm season, that rainshowers and thunderstorms require nearly twice as many gages as steady rain for a given measurement, due to the great spatial variability of convective rainfall.

To help reduce the problem of spatial variability of convective rainfall and therefore the spatial variability in the Z-R relationship, Woodley et al. (1975) studied convective type precipitation in Florida using rain gage densities near 3 km^2 per gage for ground truth verification. Conclusions were made about the nature of Florida convective showers and the optimal rain-measuring system for their area. They used a Miami Z-R relationship of $Z = 300\ R^{1.4}$ that they had developed earlier (Woodley and Herndon, 1970).

Although using rain gage networks certainly increases the confidence of direct-measurement experiments, the problems of precipitation time lag and drift exist even in the most dense rain gage networks. Other factors can also cause errors in radar rainfall estimates. A source of error in reflectivity measurement can arise from hardware calibration. Frequently, even after careful electronic system calibration, large unexplainable systematic errors in radar rainfall measurements can remain (Wilson, 1964; Harrold et al., 1974; Klazura, 1977; Saffle and Greene, 1978). Therefore it is desirable that calibration be done with rain gages.

Potentially serious sources of measurement error not associated with hardware are beam blockage by obstacles close to the radar site (Harrold and Kitchingman, 1975), anomalous propagation of the radar beam (Battan, 1973), the build-up of a precipitation film on the radome (Cohen and Tmolski, 1966; Wilson, 1978), and attenuation by precipitation (Wexler and Atlas, 1963). Attenuation effects the shorter wavelengths; therefore, it is frequently recommended that a wavelength of 10 cm be used for quantitative rainfall measurement. Microphysical and kinematic properties also influence radar estimated rainfall. Evaporation

(Atlas and Chmela, 1957), collision and coalescence (Srivastava, 1971), and drop breakup (Srivastava, 1971) all effect the drop-size distribution and therefore the radar reflectivity value, Z. Vertical motion also effects the number of drops sensed by the radar beam in a given volume. Depending on the size and number concentration of hail, and whether or not the hail is coated with water, hail can have a significant effect on signal return to the radar.

1.3 Objectives and Assumptions

The objectives of the current study are to:

1. Obtain representative Z-R relationships for selected time periods by correlating directly-measured values of Z and R using linear and non-linear regression techniques;

2. Apply the Z-R relationships to independently estimate radar rainfall amount over a given watershed area (this radar-derived amount of precipitation will then be checked against the actual amount of precipitation over the watershed area as measured by the rain gages);

3. Compare the performance of the radar-derived Z-R relationships to the relationships of Marshall-Palmer (1948) and Jones (1964) for the data sets used; and

4. Evaluate the accuracy of Z-R relationships found in precipitation cores as compared to the whole precipitation field.

In this study, it is assumed that the radar was properly calibrated for data collection in 1979 and that radar parameters (Appendix C) did not change significantly from day to day. The radar wavelength was 10 cm; this minimized rainfall attenuation of the radar signal. In addition, although the physical processes such as coalescence, evaporation and vertical motions are taking place (producing a variable drop-size distribution) while the radar is sampling volumes of drops in space, the scanning strategy used for the radar is assumed to minimize ground clutter returns and to provide accurate reflectivity values.

2. DATA AND COMPUTATIONAL METHODS

2.1 The VIN Project

During the summer of 1979, a coordinated research effort took place between the Department of Environmental Sciences of the University of Virginia, the Illinois State Water Survey (ISWS), and NOAA to investigate the role of low-level and surface convergence in the evolution of precipitating convective systems. This was known as the VIN project (Ackerman, et. al., 1983). Rain gage data was archived on magnetic tape as five-minute accumulations of precipitation for all 260 gages. The radar data was archived in the original field-recording format on magnetic tapes.

2.2 Rainfall Data

Among the field facilities in use to collect data was a network of 260 recording rain gages deployed in a quasi-uniform rectangular grid with an average gage separation of about 4.8 km with a network density of 1 gage per 23 km^2 This defines the rainfall rate grid (R-grid) and is shown in Figure 1. The instruments were standard weighing-bucket gages with 8-inch orifices and 24-hour recorder clocks. This provided resolutions of five minutes in time and 0.25 mm in rain accumulation.

Rain gage data was archived on magnetic tape as five-minute accumulations of precipitation for all 260 gages. For a specified five-minute interval, accumulation (in units of mm) recorded by each rain gage is multiplied by 12 to obtain the corresponding rain rate R (in units of mm/hr) for that gage.

Figure 1. The VIN Rain Gage Network. Solid dots represent 260 raingages. The dashed circles show range from the CHILL radar.

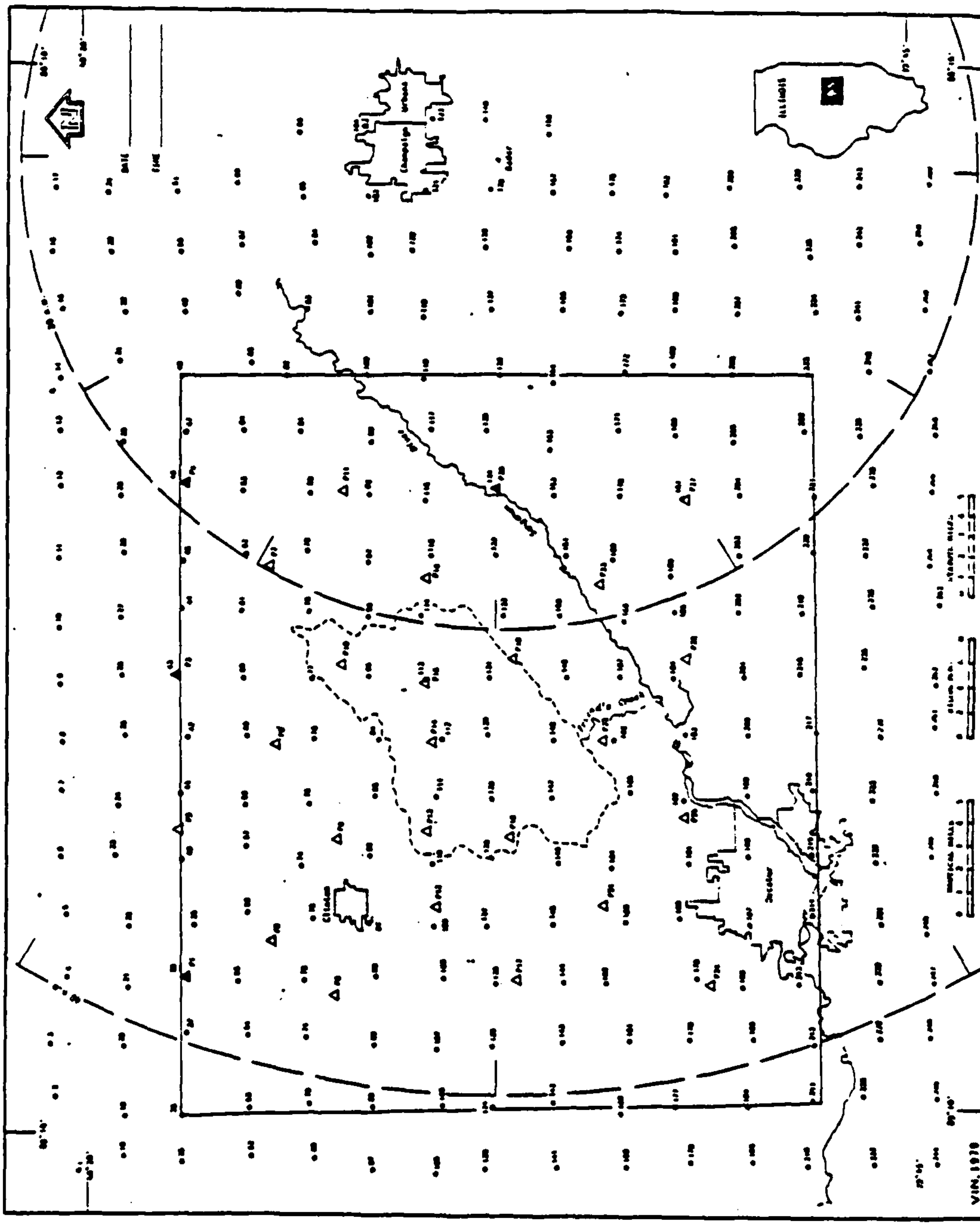

Figure 1.

Since each gage represents a 23 km^2 (9 mi^2) area, each computed rain rate for a gage is taken to be representative of a 23 km^2 area.

2.3 Reflectivity Data

The cloud volume over the rain gage network was continuously monitored by the ISWS CHILL radar. The CHILL radar was developed by the University of Chicago and the Illinois State Water Survey. During the time of the VIN project, it was a combination of 10 cm doppler and 3-cm incoherent radars which were matched with respect to pulse length, pulse repetition frequency and beam widths (1°). Both antennas were mounted on the same pedestal. The radar was located at Willard Airport in Champaign, Illinois (40°N, 88.3°W, 236 m) on the eastern edge of the rain gage network (Figure 1). The radar scanned in azimuth at a rotation rate of 16 degrees/sec. for each degree of elevation from 0.5 to 11.5 and for every 2 degrees elevation from 11.5 to 25.5 degrees. This was the general tilt sequence for the VIN project which permitted a cycle time of just under four minutes for a volume scan over the rain gage network. The maximum range of the radar was 80 nm (92 mi.). The return signals from the atmospheric targets were integrated over a time period corresponding to 150 m in range. These were recorded digitally for every range gate from the radar to the maximum range of 80 nm (92 mi.) with a one degree resolution in azimuth. The radar data was archived in the original field-recording format on magnetic tapes.

For the reflectivity values that are to be correlated with the gage rainfall rates, an averaging routine was developed to obtain a representative radar-measured reflectivity value over each rain gage in the network (Figure 2). A 40x40 horizontal x-y grid (with respect to the radar) was constructed using the elevation angles and corresponding ranges shown in Figure 2. The routine then

Elevation Angle(°)	Ranges (km)	Grid Array
0.5	40-150	First grid array
1.5	20-150	Second grid array
2.5	10-30	Second grid array
3.5	5-12	Second grid array

Figure 2. Radar Scanning Strategy to Compute Reflectivity Values (Z)

averaged all measured Z values falling within the domain of each grid point. This is done for all 1600 grid points to produce an array of reflectivity values for the .5° elevation scan. Likewise, a second reflectivity grid of 1600 values is constructed from the other three elevation angles. The two grids are then merged and the larger of the two Z values is retained on a point by point basis. The result is 1600 reflectivity values on a rectilinear grid with values separated by a distance of 2.41 km. This defines the reflectivity grid (Z-grid). The approach shown in Figure 2 gives a layered effect which minimizes the effect of ground clutter but at the same time attempts to scan as close to the gage as possible to yield an accurate radar reflectivity value.

2.4 Correlating Z and R

The next step was to correlate a given radar-measured Z value with a given gage-measured R value. The values of rain rate R are correlated with "simultaneous" radar-measured Z values for the volume above each gage to develop a Z-R relationship for the five-minute interval. Gage-measured rainrates will also be used as ground truth values to check the accuracy of directly-measured Z-R relationships (Z is determined from the radar equation which is based on the average power received from a volume of scatterers) and selected indirectly-measured Z-R relationships (Z values are computed from the measured drop-size distributions).

Since the locations of the Z-grid points and the R-grid points (rain gages) are known with respect to the radar (Z-grid origin) and the Z-grid points are separated by roughly half the distance of the rain gage separation, the four surrounding Z-values for each of 260 rain gages were determined. Then, a distance-weighted Z-value at the R-grid point was obtained from the four surrounding Z-grid points based on their distance to the gage point. (A distance-weighted-average for Z is more desirable since the rain gages are not exactly at the centers of four surrounding Z-grid points.) Now that a radar reflectivity value can be correlated with a rainfall rate value from a rain gage, a Z-R relationship can be established using statistical methods.

2.5 Statistical Methods

With Z and R now related in a data field, linear and non-linear regression techniques can be used to find a representative Z-R relationship for a specific data field.

In the linear technique (Richardson, 1944), a logarithmic transformation of the data is made and a linear regression of the transformed data is done. See Appendix D for development of this method.

If there is considerable scatter in the Z and R data, as is often the case because of the high spatial and temporal variability of precipitation, then a non-linear regression technique can be applied to reduce the statistical bias of outlier data. A system for function minimization and analysis of the parameter errors and correlations called MINUIT (James and Roos, 1977). The function to be minimized is $R=aZ^b$, with Z the independent variable. The parameters are the coefficient a and the exponent b. The equation can then be inverted to obtain $Z=AR^B$, where $A = (1/a)^{1/b}$ and $B = 1/b$. This is the form of the Z-R relationship that is most often seen in the literature.

2.6 Days Analyzed

The VIN rain gage network was in operation during July and August of 1979. The heaviest rain events occurred in the last week of July. On 24 July Tropical Storm Claudette made landfall in extreme east Texas late in the afternoon. A cold front, extending from northeast Minnesota to the Oklahoma border, moved very slowly, reaching eastern Iowa and Kansas by the end of the day. A second cold front moved into the plains and rapidly southeast, nearly overtaking the first front by midnight of July 25. See Figure 3 for the surface synoptic situation for July 24. At 500 mb, the lower pressure center associated with Claudette was in east Texas and western Louisiana. The flow was largely zonal across the northern half of the U.S. with a nearly stationary shortwave over central Iowa and Missouri. Daytime warming induced widespread convection over a large area.

Locally, in the region of the network, the wind flow was southerly with two distinct periods of rainfall. Strong outflow associated with the convective storms is also shown in Figure 4. The mid-morning rain occurred from extensive deck of strataform clouds with embedded lines of convective cells. The second period of rain in the late afternoon and evening was associated with lines of heavy thunderstorms that moved through the network.

On 25 July, Claudette had weakened and was nearly stationary over east Texas. A cold front extended from eastern Canada to west Texas in the midmorning as shown in Figure 5. The front moved eastward during the day, entering Illinois at mid-day and reaching central Illinois by midnight. Aloft, the stationary shortwave that had been in central Iowa moved east across northern Illinois during 25 July. The main rainfall on 25 July occurred in the network during the early morning hours from prefrontal lines, as shown in Figure 6. The rainfall during these two days was the most significant available during the two month operating period of the VIN project and, therefore, was used in this study.

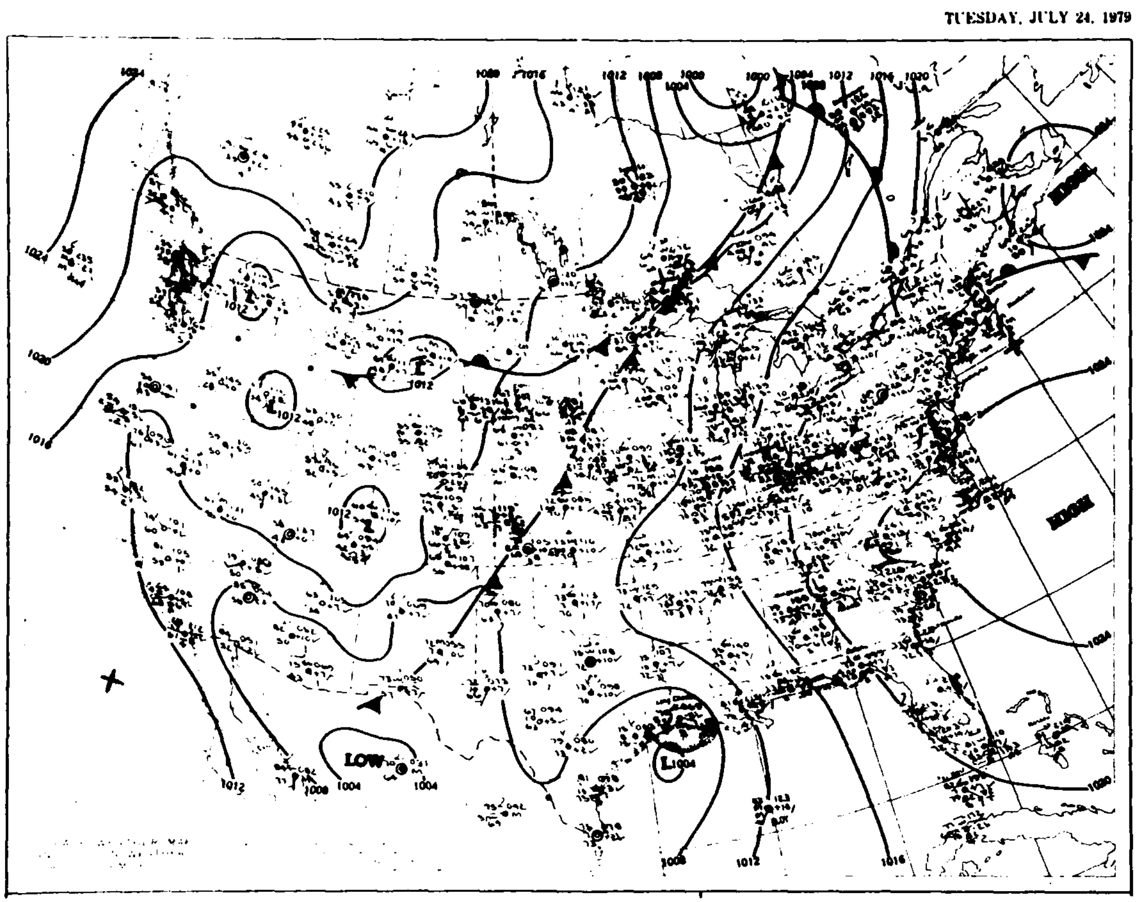

Figure 3. Synoptic Map for 24 Jul 79.

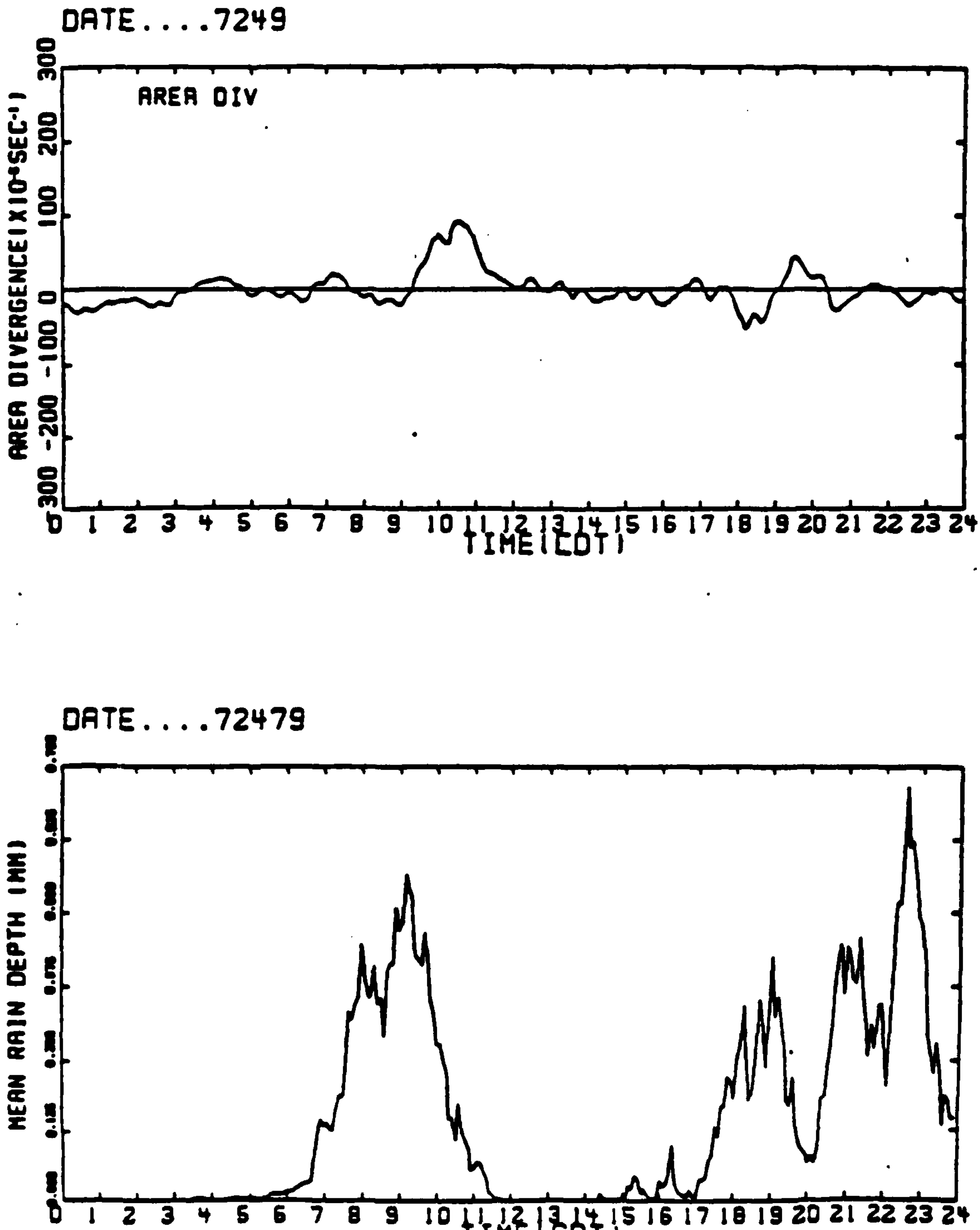

Figure 4. Precipitation Trace for 24 Jul 79.

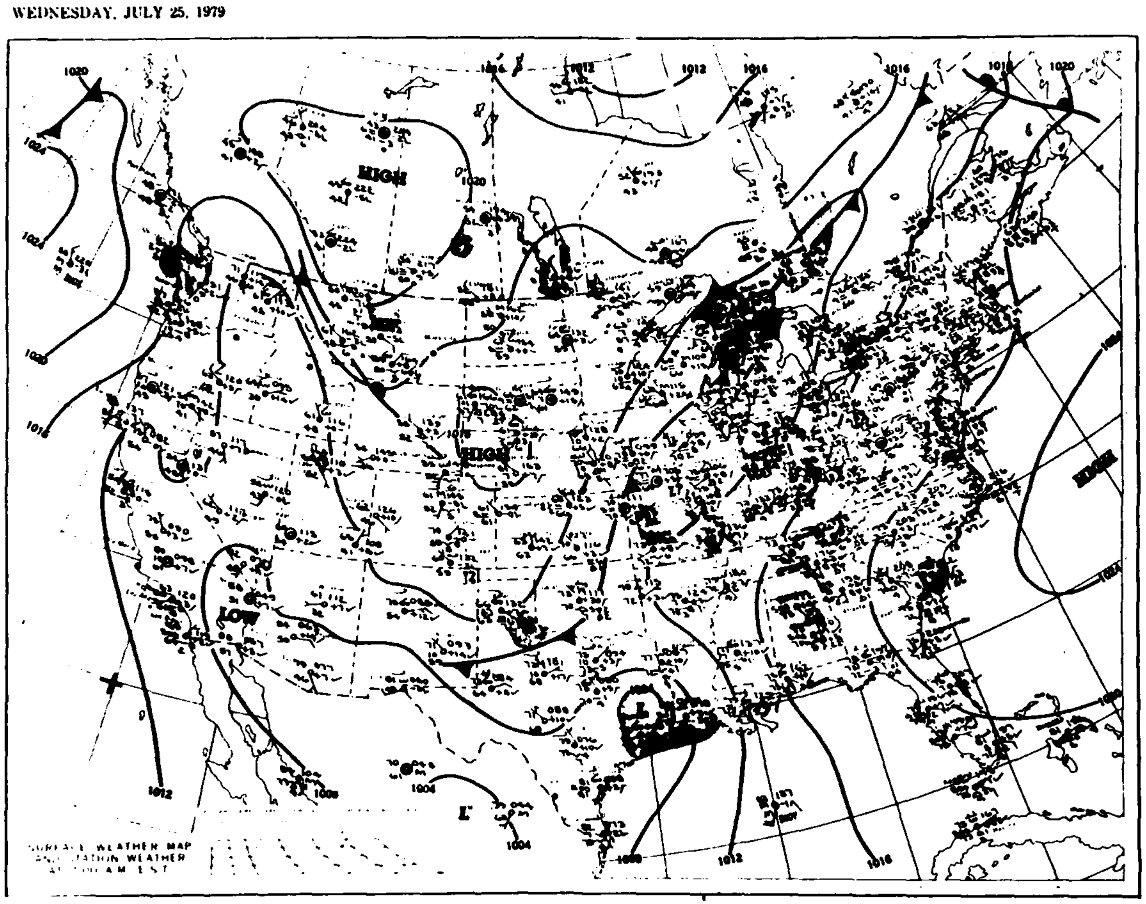

Figure 5. Synoptic Map for 25 Jul 79.

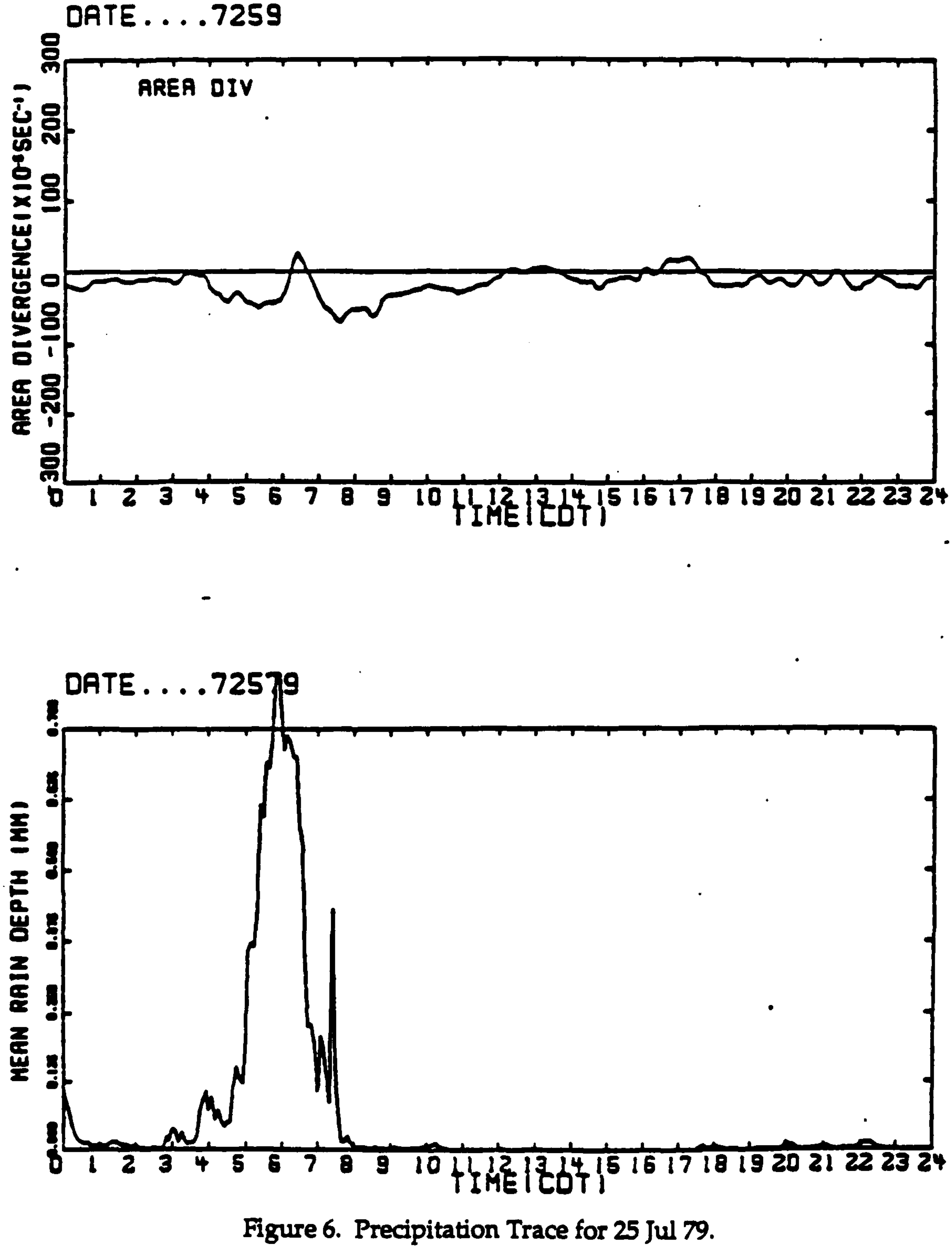

Figure 6. Precipitation Trace for 25 Jul 79.

3. RESULTS AND DISCUSSION

The purpose of this study is to obtain accurate Z-R relationships using reflectivity values directly measured by the CHILL radar and rainfall rate values measured by the VIN rain gage network for a given time period. As seen from Figure 4 and 6, these time periods correspond to peak precipitation intervals over the network. Z-R relationships have been established for the different periods using both the linear and the non-linear regression techniques discussed in Section 2.5. These relationships (together with those of other investigators) have been applied to independent data sets and the dependent data set from which they were derived to estimate the average amount of water over the gage network (watershed) area. This average amount of water for the network (as determined by a Z-R relationship) has then been compared to the actual average amount of water over the network (as measured by the gages). See Appendix E for the method of calculation of an average water amount over the network.

Table 5 shows the values of the coefficient a and the exponent b for each 5 minute period in the first data set with the range of Z greater than 15 dbz and R greater than 1 mm/hr. The values of a and b are first determined from the linear regression of $R=aZ^b$ using the logarithm of the data, log Z and log R. Then these values of a and b are used as a first approximation (first "guess") for the non-linear regression of $R=aZ^b$. This method is considered more accurate when there is large scatter in the data. Once $R=aZ^b$ is found, the expression is solved for Z to obtain $Z = AR^B$ for each 5-minute period. Similarly, Table 6 shows the results for

Table 5

Z-R Relationship for 0845-0940, 24 Jul 79
($Z \geq 15$ dbz, $R \geq 1$ mm/hr).

Time Interval	Linear ($R = aZ^b$)		Non-linear ($R = aZ^b$)		Non-linear ($Z = AR^B$)	
	a	b	a	b	A	B
0845-0850	0.667	0.328	0.581	0.294	6.34	3.4
0850-0855	0.398	0.348	0.351	0.315	27.76	3.17
0855-0900	0.188	0.460	0.118	0.484	82.72	2.07
0900-0905	0.520	0.324	0.364	0.326	22.20	3.07
0905-0910	0.425	0.355	0.547	0.267	9.58	3.75
0910-0915	0.410	0.375	0.445	0.321	12.46	3.12
0915-0920	0.294	0.394	0.361	0.332	21.52	3.01
0920-0925	0.241	0.411	0.306	0.340	32.55	2.94
0925-0930	0.313	0.371	0.198	0.379	71.74	2.64
0930-0935	0.366	0.353	0.041	0.574	261	1.74
0935-0940	0.430	0.327	0.018	0.650	483	1.54

MEDIAN VALUES

A	B
27.76	3.01

Table 6

Z-R Relationship for 0845-0940, 24 Jul 79
($Z \geq 35$ dbz, $R \geq 1$ mm/hr).

Time Interval	Linear ($R = aZ^b$)		Non-linear ($R = aZ^b$)		Non-linear ($Z = AR^B$)	
	a	b	a	b	A	B
0845-0850	5.563	0.095	0.856	.249	1.867	4.02
0850-0855	2.781	0.138	4.20	.044	6.8×10^{-15}	22.73
0855-0900	0.151	0.488	0.116	.487	83.37	2.05
0900-0905	0.532	0.329	0.043	.562	270	1.78
0905-0910	0.006	0.822	0.007	.726	929	1.38
0910-0915	0.542	0.356	3.22	.114	3.52×10^{-5}	8.77
0915-0920	0.279	0.409	2.00	.155	0.011	6.45
0920-0925	0.008	0.783	0.043	.549	308.0	1.82
0925-0930	0.036	0.609	0.0009	.946	1658	1.06
0930-0935	0.052	0.579	0.003	.833	1068	1.20
0935-0940	0.020	0.666	0.00004	1.25	3056	0.80

MEDIAN VALUES

A	B
370	1.82

the same time period for cores of precipitation where Z is greater than 35 dbz and R is equal to or greater than 1 mm/hr.

As seen from Tables 5 and 6 the coefficient A and exponent B in the relationship $Z=AR^B$ vary significantly from one five-minute period to the next. This is indicative of the spatial and temporal variation of the precipitation and, therefore, fluctuations of the rainfall rate over space and time. The median value of A and B is therefore determined to represent the entire time period and reduce the statistical effect of outlier values. Figure 7 shows a typical five-minute period of data (0915-0920, 24 July 79). The dependent variable is R along the logarithmic horizontal axis and Z is the independent variable along the logarithmic vertical axis. The data generally conforms to the Marshall-Palmer fitted line. The linear and non-linear fitted lines diverge from small values of R to large values of R. The linear least-squares method, which transforms the Z and R data to logarithmic values and fits a line to them, therefore, introduces more error for large R. Since the data sets in this study contained large values of R, the non-linear approach of fitting a curve directly to the Z and R data was used to introduce smaller error. The median Z-R expressions extracted from Tables 5 and 6, along with other established relationships, are used to calculate the average water amount over the VIN network as seen in Table 7 (Z-R relationships were computed for a given range of Z and $R \geq 1$ mm/hr while average water amounts were computed for all Z where R is measureable (i.e. $R>0$ mm/hr)).

A number of points can be suggested from the results in Table 7. First, the Z-R relationships determined from the data (0845-0940) underestimate the average water amount measured by the gages for that dependent data set by about 50%. When the "core" Z-R relationship ($Z \geq 35$dbz, $R \geq 1$ mm/hr) was used with the "core" data field ($Z \geq 35$dbz, $R>0$ mm/hr), a slightly higher percentage of

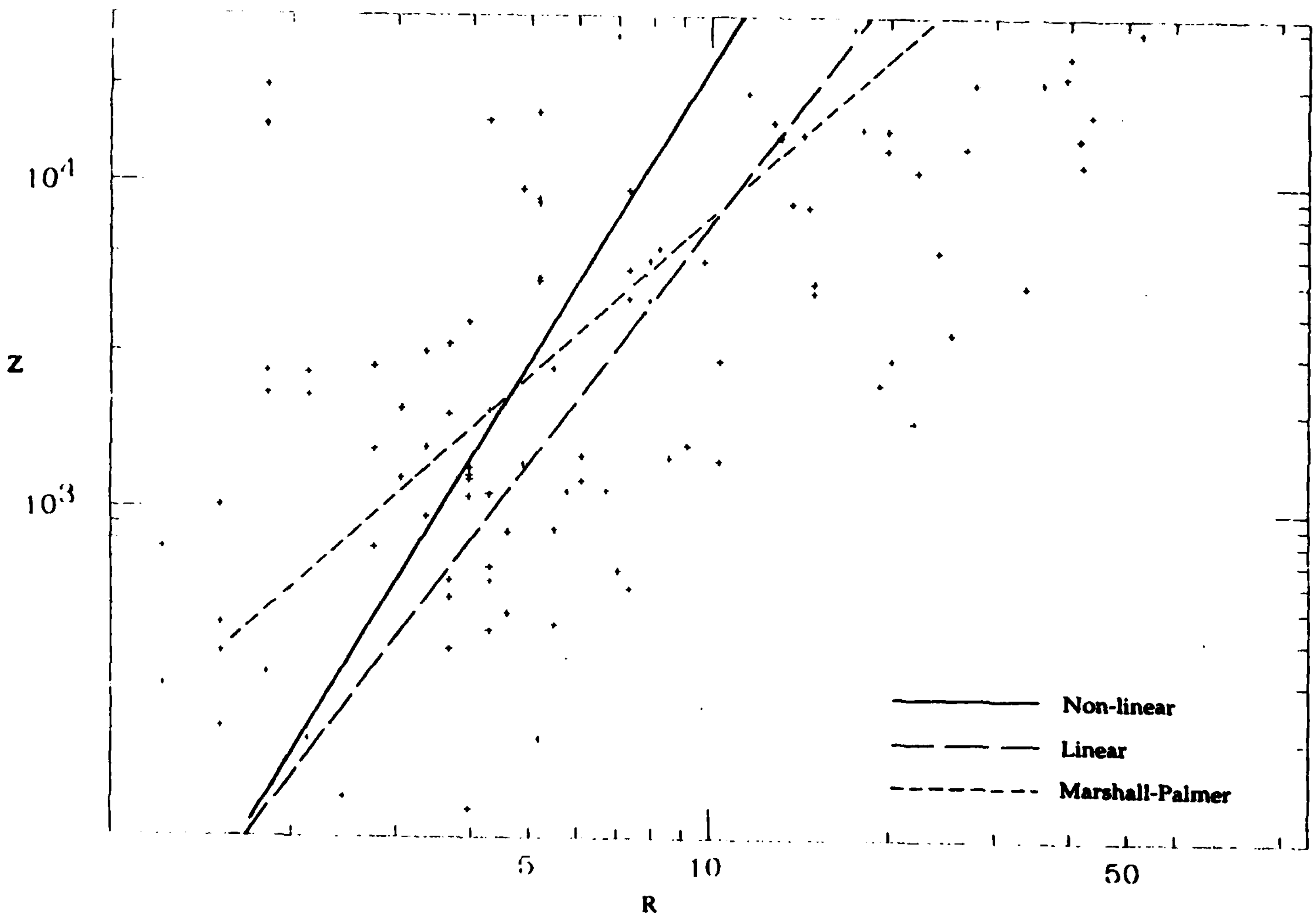

Figure 7. Scatter Diagram for a Typical Five-Minute Data Period.

Table 7

A Comparison of Z-R Average Water Amount vs. Rain Gage Average Water Amount for the VIN Network ($Z \geq 15$ dbz, $R > 0$ mm/hr).

Description	Z-R Relationship	24 Jul 79 (0845-0940)		24 Jul 79 (2200-2300)		24 Jul 79 (2300-2400)		25 Jul 79 (0500-0600)		25 Jul 79 (0600-0655)	
		Z-R (mm)	Gage (mm)	Z-R (mm)	Gage (mm)	Z-R (mm)	Gage (mm)	Z-R (mm)	Gage (mm)	Z-R (mm)	Gage (mm)
24 JUL 79 (0845-0940) $Z \geq 15$dbz $R \geq 1$ mm/hr	$Z=27.76 R^{3.01}$ $R=.331 Z^{.332}$	1.886	3.495	5.004	4.472	4.220	3.240	6.687	5.502	6.741	5.599
24 JUL 79 (0845-0940) $Z \geq 35$dbz $R \geq 1$ mm/hr	$Z=270 R^{1.82}$ $R=.046 Z^{.549}$	1.605	3.495	14.178	4.472	9.501	3.240	13.365	5.502	15.695	5.599
Marshall-Palmer	$Z=200 R^{1.6}$ $R=.0365 Z^{.625}$	2.460	3.495	34.369	4.472	21.732	3.240	29.488	5.502	35.634	5.599
Jones (1964) E. Ill. Rainshowers	$Z=370 R^{1.31}$ $R=.011 Z^{.763}$	2.509	3.495	83.021	4.472	45.873	3.240	58.972	5.502	77.413	5.599
Nexrad	$R=300 R^{1.4}$ $R=.017 Z^{.714}$	2.509	3.495	60.864	4.472	34.965	3.240	46.082	5.502	58.805	5.599
24 JUL 79 (0845-0940) $Z \geq 35$dbz $R \geq 1$ mm/hr	$Z=270 R^{1.82}$* $R=.046 Z^{.549}$	1.110	2.287	14.026	4.472	9.823	2.661	13.332	4.458	15.431	5.034

* This Z-R relationship was tested again for $Z \geq 35$dbz and $R > 0$ mm/hr.

the gage average water amount was obtained. However, applying the "whole-field" Z-R relationship ($Z = 27.76R^{3.01}$) to the "whole-field" data set ($Z \geq 15$dbz, $R > 0$ mm/hr) gave the best comparison of Z-R calculated average water amount to gage-measured average water amount.

It is suspected that the reason the Z-R relationships calculated in Tables 5 and 6 did not perform as well on their own (dependent) data set was the statistical nature in which the expressions were derived. The values of A and B are median values for the whole time period but the five-minute values of A and B varied considerably. More consistant values of A and B for each five-minute period would more than likely have yielded better results in the dependent data set of 0845-0940. It is difficult to obtain this statistical consistancy in convective-type precipitation.

Secondly, the established Z-R expressions of Marshall-Palmer, Jones, and the initial NEXRAD relationship calculated the average water amount to just over 70% of the actual amount measured by the gages for the same data set (0845-0940). Since the ranges of Z and R for which these relationships were determined are unknown, the expressions were used on the "whole-field" set of data.

Finally, the Z-R relationships calculated from the 0845-0940 data performed much better on the independent sets of data later that day on 24 July 79 and on 25 July 79. The best results show an average overestimation of 20% by $Z = 27.76Z^{3.01}$. However, the rest of the Z-R expressions overestimated the gage-measured values of average water amount for the VIN network for the independent data sets by very large amounts. These results indicated the presence of hail in the independent data sets.

Table 8

Z-R Relationship for 2200-2300, 24 Jul 79
($15dbz \leq Z \leq 55dbz$, $R \geq 1$ mm/hr)

Time Interval	Linear ($R = aZ^b$)		Non-linear ($R = aZ^b$)		Non-linear ($Z = AR^B$)	
	a	b	a	b	A	B
2200-2205	0.248	.396	.278	.314	58.600	3.18
2205-2210	0.695	.271	.0052	.733	883.800	1.29
2210-2215	0.532	.381	.598	.216	10.810	4.63
2215-2220	0.595	.297	1.402	.110	0.046	9.09
2220-2225	0.253	.402	.033	.495	983.100	2.02
2225-2230	11.987	.015	22.910	-.141	4.39×10^9	-7.09
2230-2235	0.511	.300	.038	.455	1331.900	2.20
2235-2240	1.479	.218	.294	.306	54.770	3.27
2240-2245	2.215	.206	.883	.220	1.760	4.55
2245-2250	0.400	.360	.055	.480	416.910	2.08
2250-2255	1.386	1.846	1.042	.131	0.730	7.63
2255-2300	1.644	.167	.703	.186	6.658	5.38

MEDIAN VALUES	
A	B
56.69	3.23

Table 9

Z-R Relationship for 2300-2400, 24 Jul 79
(15dbz $\leq$ Z $\leq$ 55dbz, R $\geq$ 1 mm/hr).

Time Interval	Linear ($R = aZ^b$)		Non-linear ($R = aZ^b$)		Non-linear ($Z = AR^B$)	
	a	b	a	b	A	B
2300-2305	0.556	.286	.493	.225	23.11	4.44
2305-2310	1.010	.207	.215	.291	197.88	3.44
2310-2315	0.418	.284	.180	.318	217.99	3.14
2315-2320	0.444	.259	.298	.236	169.56	4.24
2320-2325	1.054	.185	.0006	.766	16,619	1.31
2325-2330	0.384	.276	.461	.207	42.10	4.83
2330-2335	0.586	.223	.182	.292	339.28	2.46
2335-2340	0.752	.184	.178	.289	392.23	2.46
2340-2345	0.381	.298	.336	.269	57.81	3.72
2345-2350	0.491	.235	.189	.293	293.27	3.41
2350-2355	0.530	.231	.261	.263	164.73	3.80
2355-2400	1.324	.199	.138	.369	214.25	2.71

MEDIAN VALUES

A	B
206.06	3.45

Table 10

Z-R Relationship for 0500-0600, 25 Jul 79
(15 dbz $\leq$ Z $\leq$ 55 dbz, R $\geq$ 1 mm/hr).

Time Interval	Linear ($R = aZ^b$)		Non-linear ($R = aZ^b$)		Non-linear ($Z = AR^B$)	
	a	b	a	b	A	B
0500-0505	1.059	.151	0.459	.181	73.58	5.52
0505-0510	1.597	.128	0.735	.152	7.58	6.58
0510-0515	1.531	.131	0.996	.112	1.036	8.93
0515-0520	2.333	.106	0.936	.131	1.656	7.63
0520-0525	2.910	.086	1.493	.094	0.014	10.64
0525-0530	2.553	.104	1.092	.130	0.508	7.69
0530-0535	1.524	.148	0.983	.124	1.148	8.06
0535-0540	2.409	.100	1.413	.082	0.015	12.20
0540-0545	1.531	.133	1.992	.041	5.02×10^{-8}	24.39
0545-0550	0.967	.180	1.298	.085	0.047	11.76
0550-0555	0.487	.261	0.714	.159	8.32	6.29
0555-0600	0.396	.283	0.111	.340	640.84	2.94

MEDIAN VALUES	
A	B
1.092	7.87

Table 11

Z-R Relationship for 0600-0655, 25 Jul 79
(15 dbz $\leq$ Z $\leq$ 55 dbz, R $\geq$ 1 mm/hr).

Time Interval	Linear (R = aZb)		Non-linear (R = aZb)		Non-linear (Z = ARB)	
	a	b	a	b	A	B
0600-0605	0.287	.300	.383	.209	98.24	4.78
0605-0610	0.427	.274	.166	.298	417.28	3.36
0610-0615	0.323	.307	.0009	.745	12,059	1.34
0615-0620	0.172	.453	.182	.382	86.81	2.62
0620-0625	0.376	.322	.611	.201	11.63	4.98
0625-0630	0.370	.315	.606	.194	13.19	5.15
0630-0635	0.404	.294	.026	.494	1591.31	2.02
0635-0640	0.332	.301	.275	.253	163.95	3.95
0640-0645	0.175	.332	.300	.214	276.59	4.67
0645-0650	0.611	.208	.989	.089	1.132	11.23
0650-0655	1.231	.138	1.175	.066	0.087	15.15

MEDIAN VALUES

A	B
98.24	3.95

A consistant method was needed to "eliminate" the hail from the independent data sets. This was done by limiting the upper range of Z to 55 dbz, which is one of the methods used by the NEXRAD hail algorithm to determine if hail is present within a storm (NEXRAD, 1985). Once the upper limit of Z was established at 55 dbz (higher values of Z were retained when correlated with very large rainfall rates so heavy rainfall amounts would not be eliminated) and the hail was "eliminated", Z-R relationships could be established for these four hours of independent data using the same statistical methods discussed earlier. Tables 8 through 11 show the Z-R expressions calculated for each of the four time intervals. Once again, the large variability in A and B is seen over a short period of time, indicating the variable nature of convective precipitation. The performance of these Z-R relationships as well as the others are shown in Table 12 for the "no hail" data sets (15dbz $\leq$ Z $\leq$ 55dbz, R>0 mm/hr).

Examination of the results in Table 12 suggests additional points to be considered. First, the "whole-field" Z-R relationship of $27.76R^{3.01}$ once again yielded the most accurate values, two of the cases being within 10% of the gage-measured average water amount. All the average water amounts calculated by this Z-R relationship for the "no-hail" independent data sets underestimated the amount measured by the gages.

Imposing limits on the upper range of Z improved the results yielded by all the Z-R relationships. Remaining errors can be attributed to not knowing the range of the data (Z and R) in which the established Z-R expressions were determined and, even more importantly, the effect of the temporal and spatial variation of the precipitation.

Finally, although limiting Z to 55 dbz in the independent data sets improved the results, the Z-R relationships calculated from these data sets

Table 12

A Comparison of Z-R Average Water Amount vs. Rain Gage Average Water Amount for the VIN Network after Hail Removal (15dbz $\leq$ Z $\leq$ 55 dbz, R > 0 mm/hr).

Description	Z-R Relationship	24 Jul 79 (0845-0940)		24 Jul 79 (2200-2300)		24 Jul 79 (2300-2400)		25 Jul 79 (0500-0600)		25 Jul 79 (0600-0655)	
		Z-R (mm)	Gage (mm)	Z-R (mm)	Gage (mm)	Z-R (mm)	Gage (mm)	Z-R (mm)	Gage (mm)	Z-R (mm)	Gage (mm)
24 JUL 79 (0845-0940) Z$\geq$15dbz R$\geq$1 mm/hr	$Z=27.76\ R^{3.01}$ $R=.331\ Z^{.332}$			2.221	4.239	2.856	3.019	4.866	5.212	4.134	5.205
24 JUL 79 (0845-0940) 35dbz$\leq$Z$\leq$55dbz R$\geq$1 mm/hr	$Z=270\ R^{1.82}$ $R=.046\ Z^{.549}$			3.423	4.239	4.789	3.019	7.452	5.212	6.375	5.205
24 JUL 79 (2200-2300) 15dbz$\leq$Z$\leq$55dbz R$\geq$1 mm/hr	$Z=56.69\ R^{3.23}$ $R=.286\ Z^{.310}$	1.365	3.495	1.524	4.239	1.946	3.019	3.329	5.212	2.830	5.205
24 JUL 79 (2200-2400) 15dbz$\leq$Z$\leq$55dbz R$\geq$1 mm/hr	$Z=206.06\ R^{3.45}$ $R=.213\ Z^{.290}$	0.866	3.495	0.922	4.239	1.172	3.019	2.009	5.212	1.709	5.205
25 JUL 79 (0500-0600) 15dbz$\leq$Z$\leq$55dbz R$\geq$1 mm/hr	$Z=1.092\ R^{7.87}$ $R=.988\ Z^{.127}$	1.146	3.495	0.862	4.239	1.084	3.019	1.817	5.212	1.571	5.205
25 JUL 79 (0600-0655) 15dbz$\leq$Z$\leq$55dbz R$\geq$1 mm/hr	$Z=18.24\ R^{4.67}$ $R=.375\ Z^{.214}$	0.840	3.495	0.754	4.239	0.945	3.019	1.620	5.212	1.384	5.205
Marshall-Palmer	$Z=200\ R^{1.6}$ $R=.0365\ Z^{.625}$			6.659	4.239	9.592	3.019	14.201	5.512	12.214	5.205

Table 12, continued

Description	Z-R Relationship	24 Jul 79 (0845-0940)		24 Jul 79 (2200-2300)		24 Jul 79 (2300-2400)		25 Jul 79 (0500-0600)		25 Jul 79 (0600-0655)	
		Z-R (mm)	Gage (mm)	Z-R (mm)	Gage (mm)	Z-R (mm)	Gage (mm)	Z-R (mm)	Gage (mm)	Z-R (mm)	Gage (mm)
Jones (1964) E. Ill. Rainshowers	$Z=370^{1.31}$ $R=.011\,Z^{.763}$			10.946	4.239	16.488	3.019	21.885	5.512	18.995	5.205
Nexrad	$R=300\,R^{1.4}$ $R=.017\,Z^{.714}$			9.171	4.239	13.616	3.019	18.835	5.512	16.302	5.205
24 JUL 79 (0845-0940) 35dbz≤Z≤55dbz R≥1 mm/hr	$Z=270\,R^{1.82}$* $R=.046\,Z^{.549}$			3.268	3.587	4.567	2.439	7.146	4.157	6.114	4.634

* This Z-R relationship was tested again for Z≥35dbz and R>0 mm/hr.

(Tables 8 through 11) did not perform as well as those from the 0845-0940 data set. An explanation for this appears to be found in the nature of the precipitation and, therefore, the data. The precipitation from 0845 to 0940 was not as variable in space or time as that in the other time periods. Therefore, based on the results, the statistical methods used in this study appear to produce better results when large gradients in precipitation are not present over the network. Table 12 shows that $Z = 56.69R^{3.23}$ gives the best estimate of water amount with a percentage of 64% during the Z time period of 0500-0600, 25 July 79 for data sets with steep gradients of precipitation.

4. SUMMARY AND RECOMMENDATIONS

In this study, directly-measured values of reflectivity by radar and rainfall rate by rain gages were correlated over five-minute intervals to a maximum of one hour for 260 rain gage points in the VIN network. A non-linear regression technique was used to obtain a Z-R relationship for each five-minute period and a median value of the coefficient A and the exponent B was computed to yield a Z-R relationship representative of the entire hour. The average water amount for the network was computed by these "directly-measured" Z-R relationships and compared to the actual average water amount measured by the gages.

It was found that these directly-measured Z-R relationships estimated the average gage amounts much more accurately using independent data sets.

The established Z-R relationships of Marshall-Palmer, Jones, and the NEXRAD relationship performed much better for the data set that contained more gradual gradients of precipitation (0845-0940) than the later data sets with steeper gradients of precipitation. All three of the relationships estimated average water amount to within 30% of the actual amount.

Once hail was "removed" from the later data sets, estimates of rainfall amounts improved significantly. These results show the tremendous impact that hail can make in overestimating rainfall amounts by the radar and, therefore, the need for NEXRAD hail algorithm.

It was more difficult to obtain accurate Z-R expressions for the late data sets, even after the hail was "removed", because of the existence of steep gradients of precipitation. The large gradients of precipitation make it more difficult for the radar to spatially resolve a representative reflectivity value. The best estimation was about 64% for the Z-R relationships later on 24 July and 25 July.

The best results came from the Z-R relationships calculated from the 0845-0940 data set. In some cases the expressions $Z = 27.76\ R^{3.01}$ and $Z = 270\ R^{1.82}$ estimated to within 10% and 20%, respectively, of the actual average amount for the VIN network area. These results are certainly in the range of those produced by the NEXRAD hydrology sequence (Kelsch, 1988) and appear to indicate that the statistical techniques used in this study work best for the whole data field ($Z \geq 15$dbz, $R \geq 1$ mm/hr) where precipitation gradients are not as large.

However, the results are after the fact and the need exists for real-time radar estimates of rainfall amounts over an area. This is why the NEXRAD hydrology sequence is being developed and tested (Kelsch, 1988). The NEXRAD hydrology sequence is a series of algorithms that will obtain the most representative reflectivity value possible for a given gage point, calculate the rainfall rate using the relationship $Z = 300R^{1.4}$ (Kelsch, 1988), compare this radar estimate to the gage value, and make appropriate adjustments at five-minute intervals to obtain a more accurate radar estimate.

Until this technology becomes available for widespread use, the techniques used in this project offer a method for determining a Z-R relationship over a rain gage network. In particular, these methods can be applied to the CHILL radar domain for a given rain gage network. The programs would have to be modified if the reflectivity grid points, or more likely, the rain gage grid points changed. It is recommended that an ideal scanning strategy be worked out to obtain the maximum number of scans possible in a five-minute period and to maintain elevation angles that will give the most representative reflectivity values around each gage point. In this study, the maximum number of scans in a five-minute period was only two and it was assumed that the reflectivity data fields were representative of point rainfall values. Indeed, a major source of error in radar estimates of rainfall amounts is how well the radar reflectivity values represent the rain falling into the gage.

Finally, assuming that the reflectivity value measured by the radar is representative of the rainfall falling into a gage, the variable time lag in precipitation fall could also be investigated by correlating the reflectivity value measured by the radar for a five-minute period with the succeeding five-minute value of rainfall rate to see if radar estimates of rainfall amount improve.

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APPENDICES

Appendix A

The Z-R Relationship for a Marshall-Palmer Drop-Size Distribution of $n(D) = N_0 e^{-\lambda D}$ (Rodgers, 1979) and a Drop Terminal Velocity of $V_T(D) = KD^{1/2}$ (Spilhaus, 1948).

The expression for rainfall rate, R, is:

$$R = \pi/6 \int_0^{\infty} n(D) V_t(D) D^3 dD \qquad \text{(A1)}$$

where

$n(D) = Noe^{-\lambda D}$, and

$V_t(D) = KD^{1/2}$.

Here, $n(D)$ has units of mm^{-4},
λ has units of mm^{-1},
N_0 has units of mm^{-4},
K has units of $mm^{1/2}/hr$, and
V_t has units of mm/hr.

Substitution of the above parameters into Eq. A1 gives:

$$R = No(\pi/6)K \int_0^{\infty} e^{-\lambda D} D^{3.5} dD. \qquad \text{(A2)}$$

Let $x = \lambda D$ to obtain a form of the equation that one can integrate, then:

$$e^{-\lambda D} D^{3.5} dD = \frac{1}{\lambda^{4.5}} \int_0^{\infty} e^{-\lambda D} (\lambda D)^{3.5} d(\lambda D) \qquad \text{(A3)}$$

or,

$$e^{-\lambda D} D^{3.5} dD = \frac{1}{\lambda^{4.5}} \int_0^{\infty} e^{-x} x^{3.5} dx. \qquad \text{(A4)}$$

The integral in Eq. A3 can now be solved for by use of the gamma function, therefore:

$$\int_0^{\infty} e^{-x} x^{3.5} dx = \Gamma(4.5) = 11.632. \qquad \text{(A5)}$$

Substituting Eq. A5 into Eq. A2 gives the rainfall rate, R, as:

$$R = \frac{No\pi K}{6\lambda^{4.5}} (11.632), \quad \text{(with units of mm/hr)}. \qquad \text{(A6)}$$

The expression for the reflectivity, Z, is:

$$Z = \int_0^{\infty} n(D) D^6 dD \qquad \text{(A7)}$$

where

$$n(D) = N_0 e^{-\lambda D}.$$

Substitution of N(D) into (A7) gives:

$$Z = No \int_0^{\infty} e^{-\lambda D} D^6 dD. \tag{A8}$$

Similarly, let x = λD to obtain a form of the equation that can be integrated, then:

$$Z = \frac{No}{\lambda^7} \int_0^{\infty} e^{-\lambda D} (\lambda D)^6 d(\lambda D), \text{ or} \tag{A9}$$

$$Z = \frac{No}{\lambda^7} \int_0^{\infty} e^{-x} x^6 dx. \tag{Al0}$$

using the gamma function to evaluate the integral in Eq. A10 gives:

$$\int_0^{\infty} e^{-x} x^6 dx = \Gamma(7) = 6! = 720 \tag{A11}$$

where

$$Z = \frac{No\ 720}{\lambda^7} \text{ (with units of } 10^3 \text{ mm}^3\text{).} \tag{A12}$$

We now have expressions for reflectivity, Z, and rainfall rate, R. If we equate Eq. A6 and Eq. A12, substitute values for No and K, and make the appropriate unit adjustments, then with:

$N_0 = 8 \times 10^{-6}$ mm^{-4},

$K = 1.59 \times 10^7$ mm$^{1/2}$ /hr ,

$R = \frac{No \pi K}{6\lambda^{4.5}}(11.632)$,

$Z = \frac{No\ 720}{\lambda^7}$, and

so $Z = (1.845 \times 10^{-7}) R^{1.6}$ (with units of 10^3 mm^3) or,

$Z = 185 R^{1.6}$ (multiplying 10^3 mm^3 by $\left(\frac{(mm)^3}{10^{-9}\ m^3}\right)$ to obtain the units $\frac{mm^6}{m^3}$), and

$R = .038 Z^{.625}$ mm/hr.

Appendix B

Development of the Radar Equation (Hiser, 1970).

The amount of energy in a pulsed radio wave (narrow beam) intercepted by a single spherical scatterer is given by:

$$S_t = \frac{P_t G}{4\pi r^2} \tag{B1}$$

where

P_t = peak power transmitted by radar,

B = antenna gain ratio = $\frac{\text{power per unit area along beam axis}}{\text{isotropically radiated power}}$,

S_t = amount of energy intercepted by a single scatterer, and

r = range of target.

For a single spherical scatterer a back-scattering cross sectional area A_t, the power intercepted by that single scatterer is:

$$S_t = \frac{P_t G A_t}{4\pi r^2} \tag{B2}$$

The amount of energy returned to the antenna by a single scatterer of cross-sectional area A_t is:

$$Sr = \frac{S_t A_t}{4\pi r^2}. \tag{B3}$$

The power received by the receiver via the antenna is:

$$Pr = SrAe \tag{B4}$$

where Ae = the effective energy collecting area of the antenna.

Substitution of Eq. B3 into eq. B4 gives:

$$Pr = \frac{P_t G Ae A_t}{(4\pi)^2 r^2} \tag{B5}$$

where

At = $(\Sigma\sigma)$(volume illuminated), and

$\Sigma\sigma$ = total backscattering area per unit volume.

Since the precipitation drop sizes are much smaller than the 10 cm radar wavelength, Rayleigh scattering theory holds and therefore:

$$\Sigma\sigma = \frac{\pi^5 |K|^2 \Sigma D^6}{\lambda^4} \tag{B6}$$

The illuminated volume is defined by the radar beam dimensions and the pulse width and is given by:

$$V = \pi r\left(\frac{\theta}{2}\right) r\left(\frac{\emptyset}{2}\right)\left(\frac{h}{2}\right) \tag{B7}$$

where

$\frac{\theta}{2}$ = horizontal beam width,

$\frac{\emptyset}{2}$ = vertical beam width,

$\frac{h}{2}$ = width of pulse.

The backscattering cross-sectional area of a volume of scatterers is:

$$A_t = \frac{\pi^5 |K|^2 \Sigma D^6}{\lambda^4} \quad \pi r^2 \left(\frac{\theta}{2}\right)\left(\frac{\emptyset}{2}\right)\left(\frac{h}{2}\right), \quad \text{or} \tag{B8}$$

$$A_t = \frac{\pi^6 r^2 \theta \emptyset h |K|^2 \Sigma D^6}{8\lambda^4} \tag{B9}$$

where A_t is the total backscattering cross-sectional area of targets in a volume defined by beam dimensions and pulse width.

Substituting Eq. B9 into Eq. B5 gives the average power returned to the reciever by a volume of scatterers at range r as:

$$Pr = \frac{P_t A_e^2 (\pi^6 r^2 \theta \emptyset h |K|^2 \Sigma D^6)}{4\pi r^4 (8\lambda^4)}, \quad \text{or} \tag{B10}$$

$$Pr = \frac{\pi^5 P_t A_e^2 \theta \emptyset h |K|^2 \Sigma D^6}{32 r^2 \lambda^4} \tag{B11}$$

where

Pt = peak power transmitted,

Ae = effective energy-collecting area of antenna,

$|K|^2$ = refractive index parameter,

ΣD^6 = reflectivity factor (Z),

θ = horizontal beam width,

$\emptyset$ = vertical beam width,

h = pulse width,

r = range of target, and

λ = wavelength.

The quantity $\frac{\pi^5 P_t A_e^2 \theta \emptyset h}{32\lambda^4}$ is sometimes referred to as the radar constant, c.

Then Eq. B11 becomes:

$$Pr = \frac{c|K|^2 Z}{r^2} \qquad \text{(B12)}$$

Appendix C

Operational Characteristics of the Chill Radar System

Parameter	10-cm Channel	3-cm and 10-cm Channel	3-cm Channel
Antenna			
Shape	Parabolic		Polarization twist Cassegrain feed
Diameter	8.5 m		2.5 m
Half-Power Beanwidth	0.96		1.0
Gain	43.3 dB		39 dB
First Side Lobe Level	-25 dB		-30
Polarization	Horizontal & vertical on pulse to pulse basis		Horizontal
Azimuthal Antenna Rotation Rate	30°/s		Same
Antenna Controller			
PPI Capability		Yes	
Sector Scan with Variable Limits		Yes	
Azimuthal Sample Spacing		Unlimited	
Elevation Increment		Unlimited	
RHI		Yes	
Transmitter			
Wavelength	10.7 cm		3.2 cm
Frequency	2.73 GHz		9.375 GHz
Peak Power**	1 Mw		100 kw
Pulse Width	0.25, 0.5, or 1.0 μm		1 μsec (150 m)
Pulse Repetition Time-Equispaced*	800-2500 μs		1056/1230 μs
Maximum Unambiguous Range	375 Km		
Maximum Unambiguous Velocity	±34.4 m/s		
Receiver			
Noise Figure	4.0 dB		13 dB
Transfer Function	linear		logarithmic
Dynamic Range**	90 dB		55 dB
Band Width 3 dB	Varies with P.W		1.2 MHz
Min. Detectable Signal (SNR=1)**	-110		-98 dBm

* A pulse repetition staggering is possible permitting larger unambiguous ranges.

** Representative value.

Parameter	10-cm Channel	3-cm and 10-cm Channel	3-cm Channel
Data Acquisition			
No. of Range Gates	1024-4096		1024-4096
Range Gate Spacing	0.25, 0.5, 1.0 μs		1 μs
Recorded Word Length			
Velocity	8 bits (2's comp)		
Width	8 bits (binary)		
Intensity	8 bits (binary)		8 bits (binary)
Ground Clutter Canceller	Not decided		No
Number of Samples in Estimate	Arbitrary		Arbitrary
Tape Recording			
Format		Almost Universal Recording	
Tape Density		6250 cpi	
Block Length		≤ 8192	
Initial Variables Available ***			
Reflectivity	Yes		Yes
Horizontal Polarization	Yes		Yes
Vertical Polarization	Yes		No
Cross Polarization****	Yes		No
Differential	Yes		No
Velocity (from pulse pair algorithm)	Yes		No
Width (from 2nd lag pulse pair algorithm)	Yes		No
Correlation function with lags of 1	Yes		No
Normalized Coherent power	Yes		No
Doppler Spectra from FFT processing	Yes		No

*** Other variables or variants of these variables can be obtained by reprogramming of the preprocessor.

**** With accuracy reservations.

Appendix D

Least-Squares Method of Obtaining an Equation of the Form $y = Ax^B$ using a Logarithmic Transformation of the Data.

The objectives are to obtain the equation of the line that fits logarithmically transformed data and the equation of the curve that fits the original data using a linear least-squares technique. The independent variable is x.

Table D1. Sample Data for Linear Least-Squares Technique.

n	x	y	logx	logy	(logx)(logy)	$(\log x)^2$
1	4	2.9	.602	.462	.278	.362
2	8	23.0	.903	1.362	1.230	.815
3	12	77.8	1.079	1.891	2.040	1.164
4	16	184	1.204	2.265	2.727	1.450
5	20	360	1.301	2.556	3.325	1.692
6	24	622	1.380	2.794	3.855	1.904

In general, if $y=Ax^B$, then

$\log y = \log A + B \log x$, or letting $X = \log x$ and $Y = \log y$,

$Y = a + BX.$

To calculate the slope of the fitted line $Y = a+BX$:

$$\text{slope} = B = \frac{n\Sigma \log x \log y - \Sigma \log x \Sigma \log y}{n\Sigma(\log x)^2 - (\Sigma \log x)^2}, \text{ or}$$

slope = B = 2.97.

To calculate the intercept of the fitted line $Y = a+BX$:

$$\text{intercept} = a = \frac{\Sigma(\log x)^2 \Sigma \log y - \Sigma \log x \Sigma(\log x)(\log y)}{n\Sigma(\log x)^2 - (\Sigma \log x)^2}, \text{ or}$$

intercept = a = -1.32.

The line that fits the logx and logy data is now:

$$Y = -1.32 + 2.97X$$

where $a = \log A = -1.32$
$A = 0.048$
$B = 2.97.$

Therefore, the curve that fits the x and y data is:

$y = Ax^B$, or

$y = .048x^{2.97}.$

Appendix E

Calculation of Average Rainfall Amount Over the Rain Gage Network.

The total amount of water calculated for all gages as calculated by a Z-R relationship is:

$$W_T = \frac{\sum_{i}^{N} \sum_{t} Rit}{12}$$

where

i = number of gages,

N = total number of gages,

t = 5 minutes,

R_{it} = rainfall rate calculated by the Z-R relationship for a 5-minute period, t, for a gage, i, and

W_T = total amount of water calculated by a Z-R relationship for all gages in the network.

The average amount of water over the rain gage network is:

$$W_{ave} = W_T/N.$$

The total amount of water for all gages as measured by the gages in the network is:

$$W_T = \frac{\sum_{i}^{N} \sum_{t} Rit}{12}$$

where

i = number of gages,

N = total number of gages,

t = 5 minutes,

R_{it} = rainfall rate as measured by the gage, i, for a 5-minute period, t, and

W_T = total amount of water measured by all gages in the network.

The average amount of water over the network is:

$$W_{ave} = W_T/N.$$